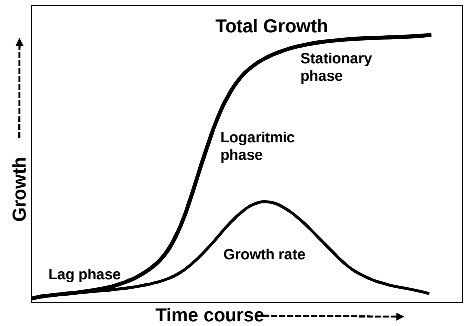
LECTURE 09
EMPIRICAL MODELS



Why the accumulation of plant biomass with time
1. increases slowly at the beginning
2. increases rapidly thereafter, and
3. is almost constant at the end of growing period

LEARNING OUTCOMES

After the completion of this lecture and mastering the lecture materials, students should be able

1. to apply several empirical models to analyze the data of plant growth
2. to decide what empirical models to be used to obtain information from the data of plant growth

LECTURE OUTLINE

- INTRODUCTION
- INITIAL PLANT GROWTH ANALYSIS
  - **'Classical' Approach**
  - 'Functional' Approach
- EMPIRICAL MODELS
  1. Linear Model
  2. Exponential Model
  3. Power (Allometric) Model
  4. Polynomial Model
  5. Logarithmic Model

INTRODUCTION

What is an empirical model?

1. Many problems in engineering and science involve exploring the relationships between two or more variables
2. For example, in a chemical process, suppose that the yield of the product is related to the process-operating temperature
3. An empirical model is based only on data and is used to predict, not to explain, a system
4. An empirical model consists of **a function** that captures **the trend of the data**

5. An empirical model is a model where the structure is determined by the observed relationship among experimental data.
   - These models can be used to develop relationships for forecasting and describing trends.
   - These relationships and trends are not necessarily mechanistically relevant.
     - Source: EPA website, glossary of frequently used modeling terms.

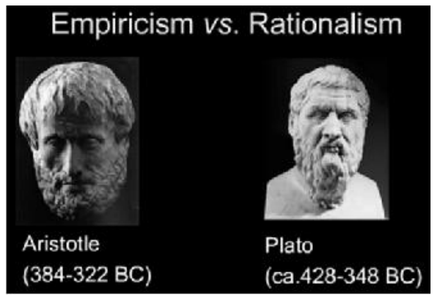
6. *Models are only useful if they help you solve problems*.
7. The best model, whether it is theoretical or empirical, is the model that predicts best for your situation

8. An Example of an Empirical Model:
   - Investigating the relationship of inflowing nutrients in a lake to algal biomass production (eutrophication).
     - Most early (circa 1970) lake eutrophication models based on statistical relationships between mass loading of nutrients and average algal biomass (e.g., Vollenweider models with numerous adaptations by others)
     - Applied to PL-566 reservoirs in North Bosque River Watershed

Where does human knowledge ultimately come from?

- Empiricists have always claimed that sense experience is the ultimate starting point for all our knowledge.
- Rationalists have claimed that reason, *not the senses*, is the ultimate starting point for all knowledge.

- Empiricists stress **induction** which involves generalizing from observables (*Specific*→*General*).
  - Science
- Rationalists stress **deduction** which involves inferring from first principles (*General* →*Specific*).
  - Mathematics



INITIAL PLANT GROWTH ANALYSIS

A. 'Classical' Approach

1. The **'classical' approach is o**ne of the oldest methods in plant growth analysis
2. This classical approach known as RGR (relatie growth rate) was introduced in the beginning of 20th century (Blackman 1919, West *et al*. 1920)
3. This is calculated by dividing the difference in "*ln-transformed plant weight* "at two harvests by the time difference between those harvests.

$$\mathbf{RGR} = \frac{\ln \mathbf{W}_2 - \ln \mathbf{W}_1}{\mathbf{t}_2 - \mathbf{t}_1}$$

4. The RGR is derived from the exponential model on the basis of an assumption that

"an increase in plant biomass (ΔW or ∂W) over a short period (Δt or ∂t) is determined by the size of plant" as follows

$$\frac{\delta W}{\delta t} = rW \qquad \frac{\delta W}{W} = r\delta t \qquad \int_{W_1}^{W_2} \frac{\delta W}{W} = r\int_{t_1}^{t_2} \delta t$$

$$\ln W_2 - \ln W_1 = r(t_2 - t_1)$$

$$RGR = r = \frac{\ln W_2 - \ln W_1}{t_2 - t_1} \qquad \int_{W_0}^{W_t} \frac{\delta W}{W} = r\int_{0}^{t} \delta t$$

5. Therefore, the use of RGR applies to plant growth which follow exponential pattern

$$W_t = W_0 e^{rt}$$

6. This approach, although straightforward, has been considered **unsatisfactory** based on the following reasons
   a. The accumulation of plant biomass (W) with time **may be not exponential** as the base of RGR
   b. The time course of the growth rate may be severely obscured by **fluctuations in RGR** between adjacent harvest intervals (Causton and Venus 1981)
   c. In the calculation of the net assimilation rate (NAR), **a fixed relation between leaf area and plant weight has to be assumed** (Evans 1972)
   d. **It is difficult to statistically evaluate differences in RGR** (Poorter and Lewis 1986).

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B. ‘Functional' Approach

1. The 'functional' approach, developed in the 1960's (Vernon and Allison 1963, Hughes and Freeman 1967), has been presented as a solution to the previous RGR problems
2. The approach proposed a polynomial of the form

$$Y = b0 + b1X + b2X^2 + ... + bnX^n$$

to analyze the growth data where **Y** = **the ln-transformed weight of the plant**, and **X** = the time

3. By differentiating this equation, an equation for RGR is obtained

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4. Hunt (1982) mentions 12 advantages of this method namely, among others,
   (1) The functional approach provides a clearer perception of ontogenetic drift;
   (2) Assumptions involved in the calculation of mean values of NAR are avoided;
   (3) Statistical analyses may be integrated into the same analytical procedure as the calculation of the derived quantities.
5. However, the functional approach does not necessarily result in correct values for RGR, NAR and the confidence limits of these parameters.

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6. Poorter and Lewis (1986) showed that the testing of differences in RGR had only limited biological meaning.
7. Wickens and Cheeseman (1988) argued that the functional approach is of limited value if plants are subjected to short-term environmental changes.
8. But even when plants are grown in a constant environment, the functional approach has some pitfalls.
9. The main problem of the functional approach is the choice of the appropriate degree of the polynomial to fit the data

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- Selecting a first degree polynomial will inevitably result in a constant RGR, whereas
- a quadratic function will invariably lead to a linearly increasing or decreasing value of RGR with lime.

10. This implies that complex growth patterns are 'underfitted' if a too low order polynomial is used.
11. Hughes and Freeman (1967) proposed fitting all data with a third degree polynomial to be able to describe both simple and more complicated growth curves.
12. However, Nicholls and Calder (1973) showed that a high degree of the polynomial may lead to 'overfitting', resulting in an RGR with spurious upward or downward trends, especially at the ends of the curve.

EMPIRICAL MODELS

1. The analysis of plant growth with the empirical models (correlative or statistical models) is the use of available models to describe relationships among variables without referring to the correlated processes
1. The empirical models are not derived from assumptions concerning the relationship between variables and they are not based on physical principles
2. The first step in deriving an empirical models is to get the scatter plot of the data.

3. If the data does not seem to be linear, try to plot one or both variables (X & Y) as logarithms so that you can check if an exponential or power models are good fits.
4. The idea is to get a graph that looks reasonably linear and then to get a linear model
5. Models available in excel program that can be used directly are→
   a. Linear Model
   b. Exponential Model
   c. Power (Allometric) Model
   d. Polynomial Model
   e. Logarithmic Model

- Keep in mind that:
  - In Linear model, y depends linearly on x

    $y = ax + b$
  - In Power model, ln y depends linearly on ln x

    $y = ax^b \rightarrow \ln y = \ln a + b \ln x$
  - In Exponential model, ln y depends linearly on x

    $y = ae^{bx} \rightarrow \ln y = \ln a + bx$
  - In Logarithmic model, y depends linearly on ln x

    $y = a \ln(x) + b \rightarrow x = e^{[(y-b)/a]}$

The following procedure could be used for the application of empirical modeling to plant growth

(1) Collect data

(2) Open excel program, and type the data (columns and rows)

I. First Approach of Analysis

(1) Block the data (highlight), click insert menu, then click scatter, and choose (click) the graph you like

(2) Set the performance of the graph to look it better

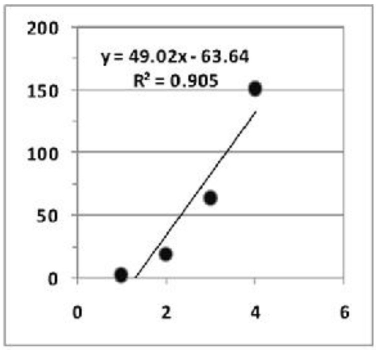
II. Second Approach of Analysis

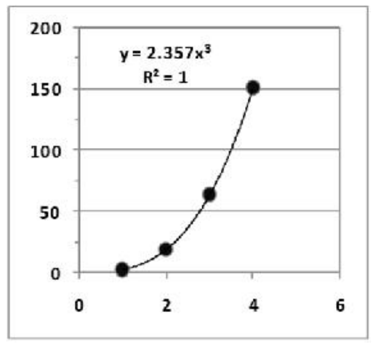
(1) Apply analysis with empirical models by clicking the data in the figure, then click layout, trendline, and choose (click) a model you like.

(2) if model is not satisfactory, click the trendline and delete it, then go to the initial step

Example 1: The relation between the radius (**r**) and the volume (**V**) of a sphere (bola) are measured as table below indicates.

r	1	2	3	4
V	2.357	18.857	63.643	150.857



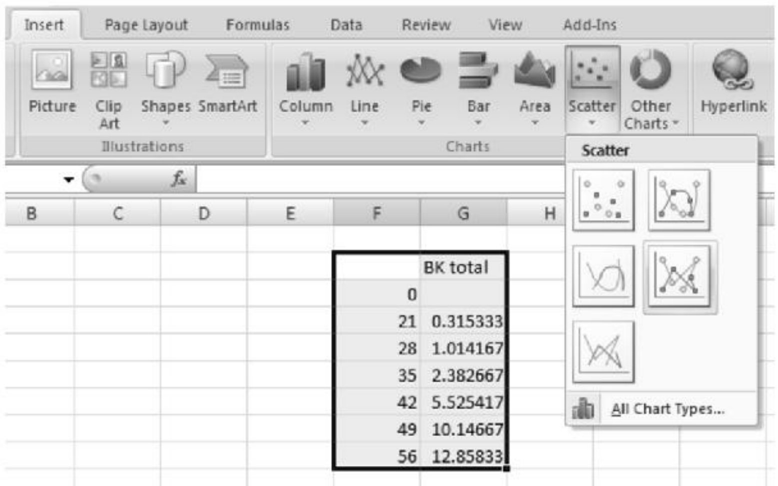


- The relationship between the radius **r** (x) and the volume **V** (y) of a sphere is

  V = 4.1905r^3
- We know that $V = \frac{4}{3}\pi r^3$

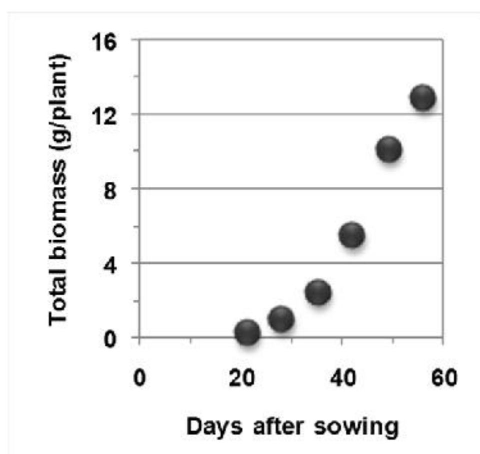
  and r = circumfernce (C)/diameter (D) = 22/7
- So 4.1905 = 4/3*π

Example 2; Plant Growth



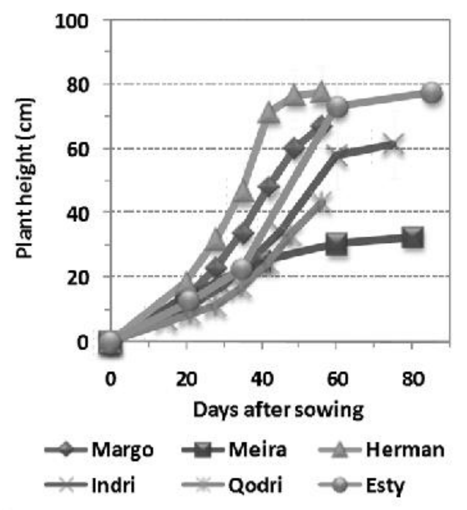
I. The first approach of analysis

This first approach is aimed at understanding the general trend of data



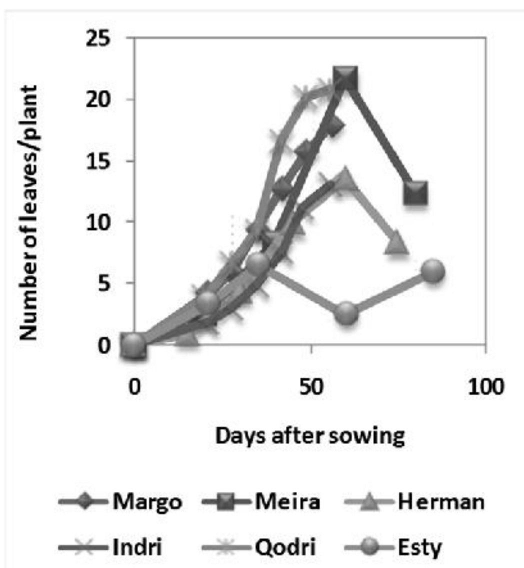
This can be helped by applying the line

Plant height

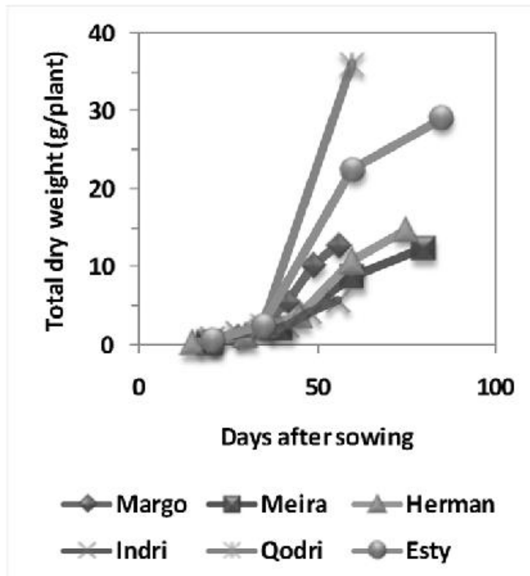


Plant height vs Time (Soybean)

Number of leaves

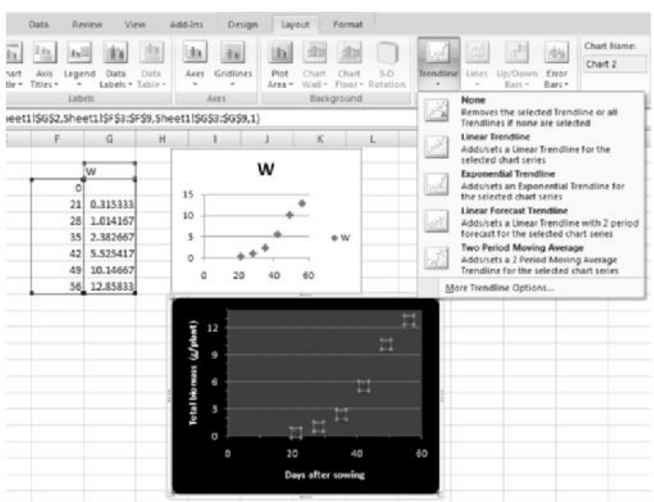


Total dry weight (W)

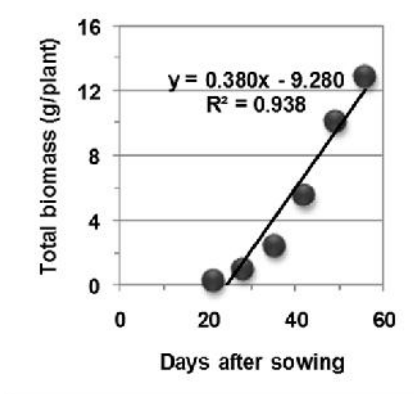


II. The second approach of analysis

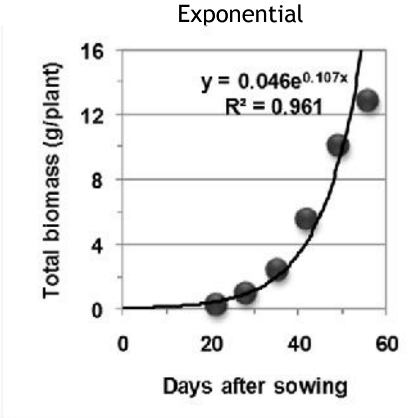
This second approach is the application of empirical models



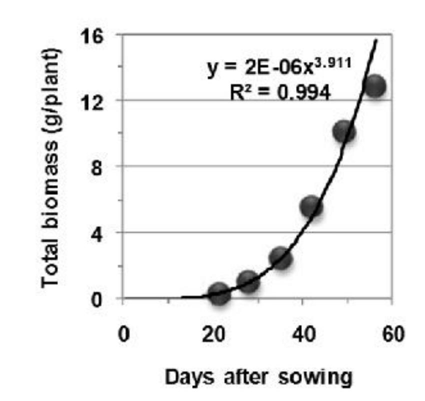
Linear



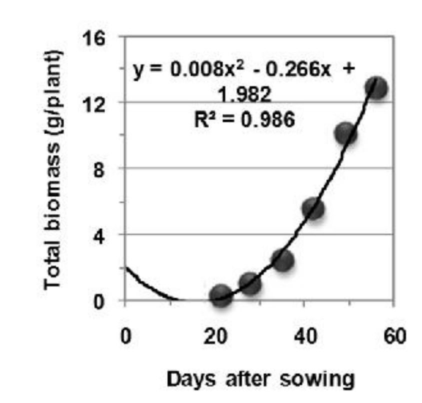
Exponential



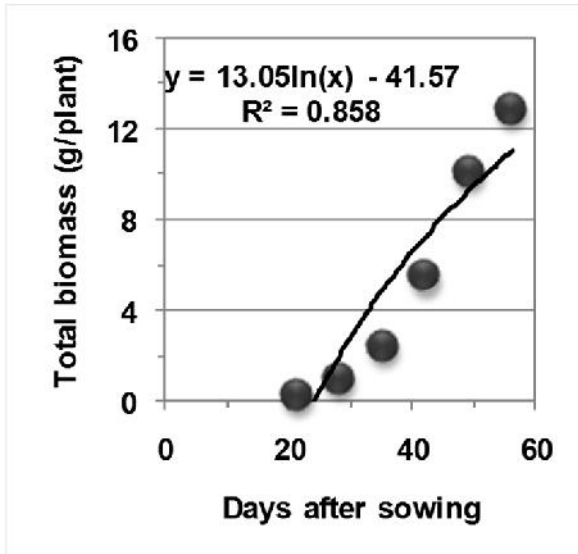
Power (Allometric)



Polynomial

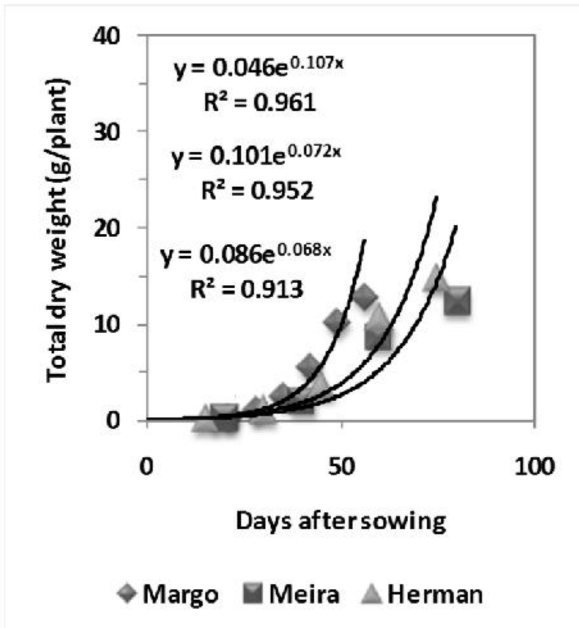


Logarithmic

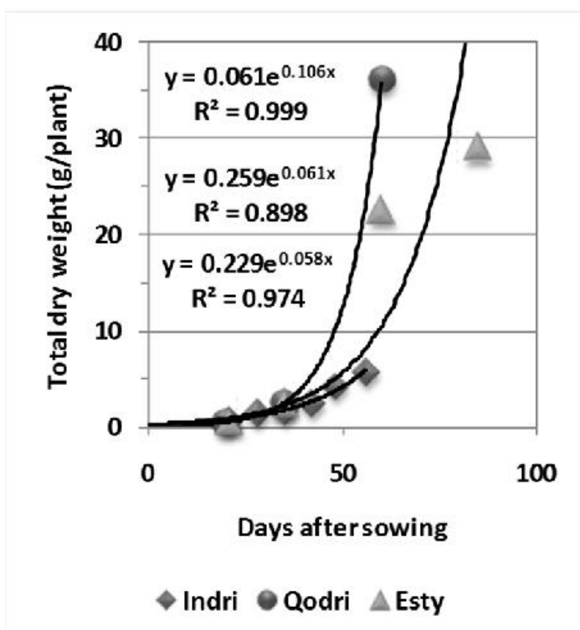


Which one you choose

- Linear $R^2 = 0.938$
- Exponential $R^2 = 0.961$
- Power (Allometric) $R^2 = 0.994$
- Polynomial (quadratic) $R^2 = 0.986$
- Logarithmic $R^2 = 0.858$

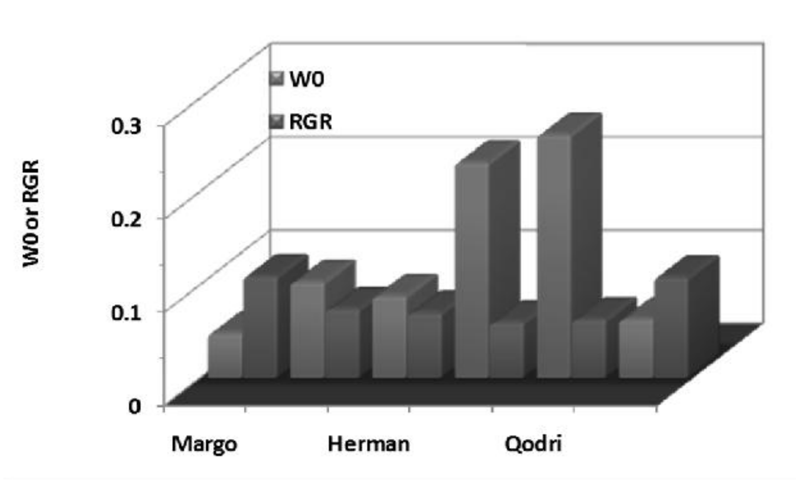


Soybean



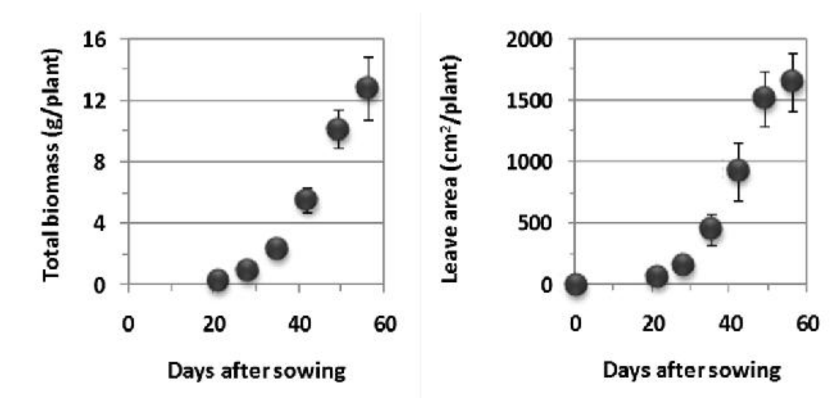
Soybean

Soybean



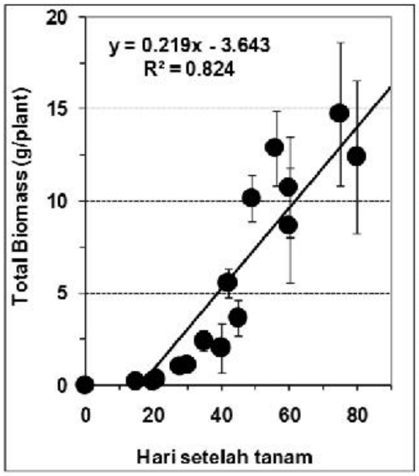
Soybean

The application of bars (standard deviation of data from replicates)

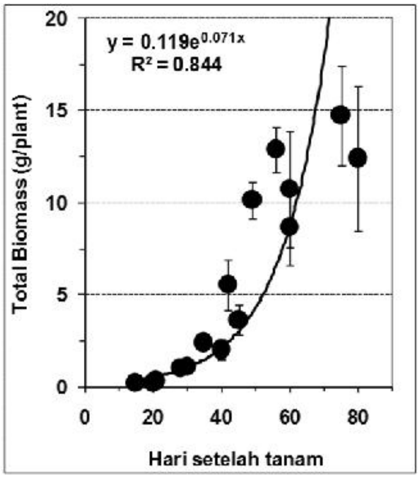


Soybean

Linear model

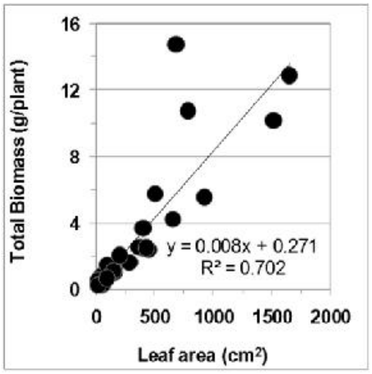


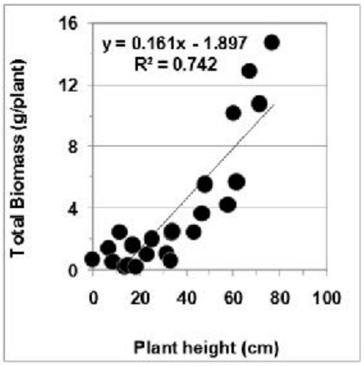
Exponetial model

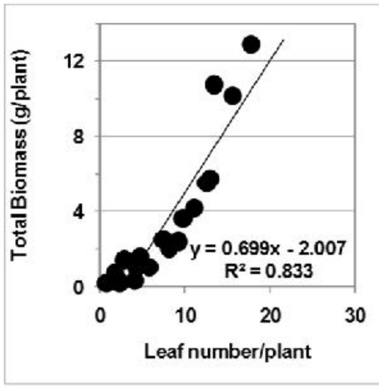


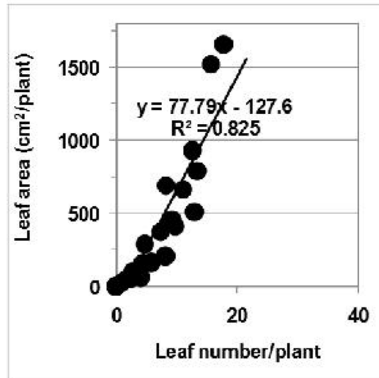
III. Other analysis

This analysis is applied to understand the relationship between the variables of plant growth



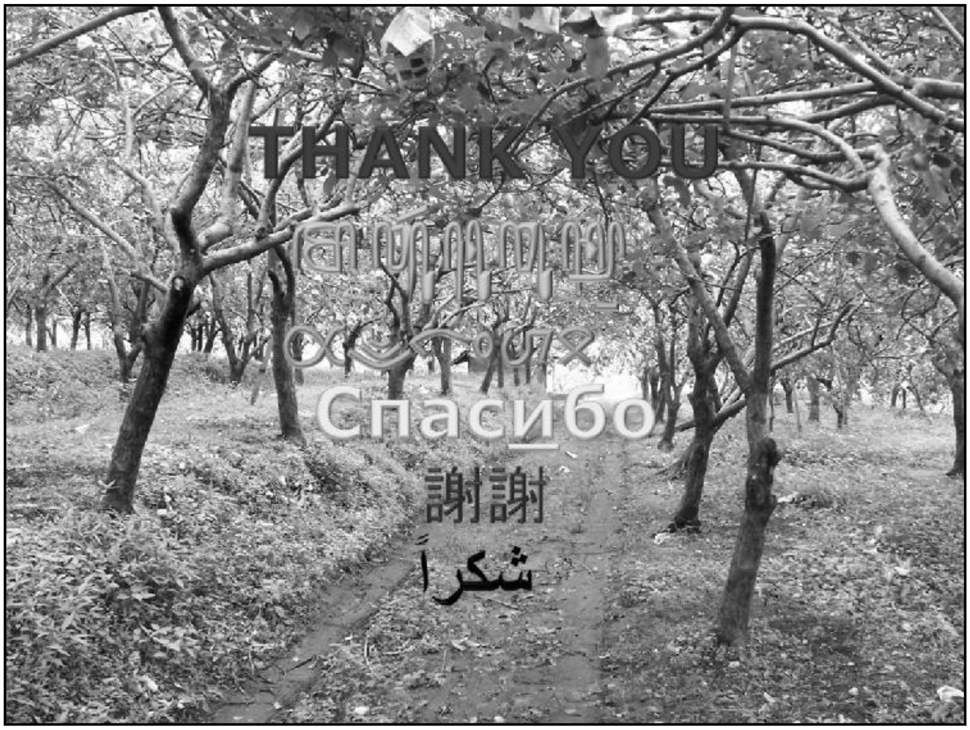






WHAT DO YOU GET?

- The application of empirical models to plant growth only give the description of plant growth
  - Total dry weight, plant height, leaf area or number of leaves increased linearly with time
  - A close relationship was found between total dry weight and leaf area, plant height or number of leaves



No.	Why a certain thing happens (What is it)	How a system works
1.	A cup of hot coffee cools down with time (why) $\rightarrow$ Ok & easy to do	
2.	Air temperature under a leaf is lower than that at open space on sunny days $\rightarrow$ OK & easy to do	
3.	Healthy leaves look green (why) $\rightarrow$ OK, but not easy to do	
4.	Some fruits look yellow (why) $\rightarrow$ OK, but not easy to do	
5.	The angle between a branch and stem seems smaller as the ratio of branch/stem diameter increases $\rightarrow$ OK & easy to do	
6.	An egg cannot stand vertically $\rightarrow$ OK & easy to do?	
7.	s $\rightarrow$ OK & easy to do	
8.	s $\rightarrow$ OK & easy to do	
9.	s $\rightarrow$ OK & easy to do	
10.	s $\rightarrow$ OK & easy to do	