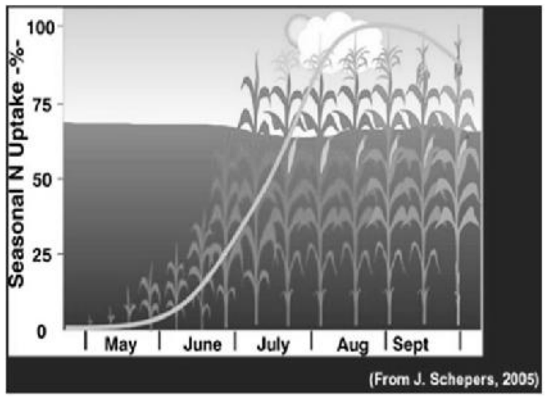
LECTURE 12: APPLICATION OF PLANT GROWTH MODEL



Model Application

How To Use ?

- Monomolecular Model

  W = Wmax(1-e^{rt})

  This model is not available in excel for data analysis

  Linearization of the model is an alternative approach

  - W = Wmax(1-e^{rt})$\rightarrow$W/Wmax = (1-e^{rt})
  - (1- W/Wmax) = e^{rt} $\rightarrow$ use exponential model and set intercept = 1 or
  - ln(1- W/Wmax) = rt $\rightarrow$ use linear equation (y = ax + b) by setting b = 0

LECTURE OUTCOMES

Students, after mastering materials of the present lecture, should be able

1. to apply logistic, Gompertz, and Richards models in the analysis of plant growth
2. to explain the base of leaf unit rate of net assimilation rate (NAR)

LECTURE OUTLINE

1. PLANT GROWTH MODELS
   1. Logistic Model: $W = \dfrac{W_m}{1 + \alpha e^{-\mu t}}$
   2. Gompertz Model: $W = W_0 e^{\left[\frac{\mu_0}{D}\left(1 - e^{-Dt}\right)\right]}$
   3. Richards Model: $W = \dfrac{W_0 W_m}{\left[W_0^n + \left(W_m^n + W_0^n\right)e^{-kt}\right]^{1/n}}$
   4. Chanter Model: $W = \dfrac{B}{1 + \left(B/W_0 - 1\right)\exp\left[-\mu\left(1 - e^{-Dt}\right)/D\right]}$

2. W AND LA RELATION

1. PLANT GROWTH MODEL

1. Logistic Model

$$W = \frac{W_m}{1+\alpha e^{-\mu t}}$$

where Wmax = asymptotic max, α & μ = shape parameters

- This was developed by P.F.Verhulst in1838 as a model for population growth.
- Often used for sigmoid growth where the inflection is located at approximately half of the ultimate value.

Ref:Verhulst,P.F.(1838):*Noticesurlaloiquelapopulationsuitdanssonaccroissement.Corr. Math.Phys.10,113-121*

- Linearization that is the rearrangement of the model to a linear form (y = ax + b) is an alternative, simple approach in the application of the model.

$$W = \frac{W_m}{1+\alpha e^{-\mu t}}$$

$Wm/W = 1+\alpha e^{-\mu t} \rightarrow 1-(Wm/W) = \alpha e^{-\mu t}$

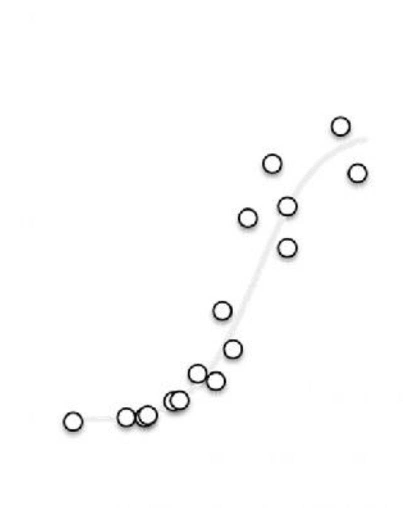
$\ln[1-(Wm/W)] = \ln\alpha - \mu t \rightarrow y = ax + b$

$y = \ln[1-(Wm/W)]$, $b = \ln\alpha$, $a = \mu$, and $x = t$

- The value of Wm has to be determined first and can be changed during the analysis to give the best result

$$W = \frac{W_m}{1 + \alpha e^{-\mu t}}$$

The application of the model to soybean data



2. Gompertz curves

$$W = W_0 e^{\left[\frac{\mu_0}{D}\left(1-e^{-Dt}\right)\right]}$$

W=W$_0$.exp[-«$_0$/D*(1-exp(-Dt)]

where W$_0$ = initial biomass, «$_0$ & D = shape parameters

- This was developed by B. Gompertz in 1825 for the calculation of mortality rates.
- This model results a slow growth at start and end

Ref:Gompertz,B.(1825):*Onthenatureofthefunctionexpressiveofthelawofhumanmortality,andanewmodeofdeterminingthevalueoflivecontengencies,Phil.Trans.Roy.Soc.182,513-585*

- This is one of the most frequently used models in growth mathematics.
- The approach of linearization is not applicable to this model as shown below.

$$W = W_0 e^{\left[\frac{\mu_0}{D}\left(1-e^{-Dt}\right)\right]}$$

W/W$_0$ = exp[-μ_0/D*(1-exp(-Dt))]

ln(W/W$_0$) = -μ_0/D*(1-exp(-Dt)

- The fine-tuning approach, after an initial value of μ_0 and D being determined arbitrarily, to obtain the best result is an alternative way.

- The other form of Gompertz model can be rearranged to a linear forma as follows.

$$\mathbf{W} = \mathbf{A.e}^{\left(-\mathbf{b.e}^{(-\mathbf{ct})}\right)} \qquad \mathbf{W} = \mathbf{W}_0\mathbf{e}^{\left[\frac{\mu_0}{\mathbf{D}}\left(1-\mathbf{e}^{-\mathbf{D}t}\right)\right]}$$

W/A = exp[-b(exp(-ct))]

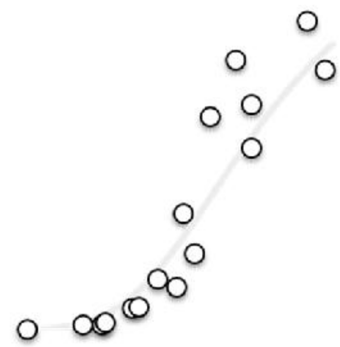
ln(W/A) = -b(exp(-ct))

ln(ln(W/A)*-1) = ln(b) -ct

The last equation can be analyzed with a linier equation (y = ax + b)

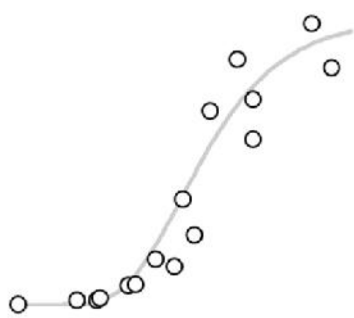
y = ln(ln(W/A)*-1), a = c, b = ln(b) and t = x

The application of the model to soybean data



$$W = A.e^{\left(-b.e^{(-ct)}\right)}$$

Wmax	15
b	-18.7583
c	-0.0705



3. Richards models

$$W = \frac{W_0 W_m}{\left[W_0^n + \left(W_m^n + W_0^n\right)e^{-kt}\right]^{1/n}}$$

- Developed by F.J. Richards in 1959 as a generalization of classical growth models.
- Brody, Bertalanffy, Gompertz and logistic models are all special cases of the Richards model.
- The linear form of Richards model is as follows

  $W=W_0W_m/(W_0^n+(W_m^n+W_0^n)e^{-kt})^{1/n}$

  $W_0W_m/W= (W_0^n+(W_m^n+W_0^n)e^{-kt})^{1/n}$

  $(W_0W_m/W)^n = W_0^n+(W_m^n+W_0^n)e^{-kt}$

Ref:Richards,F.J.(1959):*Aflexiblegrowthcurveforempiricaluse.J.Exp.Bot.10,290-300*

$(W_0W_m/W)^n - W_0^n = (W_m^n+W_0^n)e^{-kt}$

$\ln((W_0W_m/W)^n - W_0^n) = -kt + (W_m^n+W_0^n)$

- The above equation can be analyzed with a linear equation where

  $y = \ln((W_0W_m/W)^n - W_0^n)$

  $a = -k$

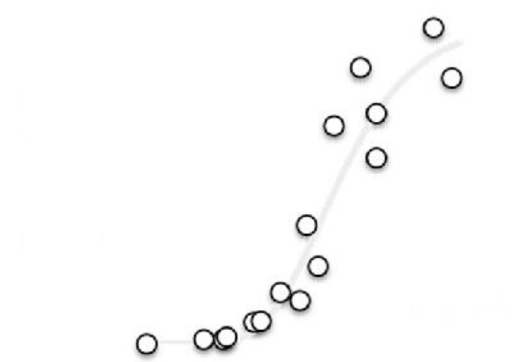
  $b = (W_m^n+W_0^n)$

  $t = x$
- Other form of Richards model is

  $W=A(1-b*\exp(-ct))^M$

Ref:Richards,F.J.(1959):*Aflexiblegrowthcurveforempiricaluse.J.Exp.Bot.10,290-300*

The application of the model to soybean data



2. W AND LA RELATION

- The biomass of plants is dominated by the product of photosynthetic process, and leaves are the main photosynthetic organ of plants.
- This fact may lead to an assumption that the production of biomass per unit time ($\partial W/\partial t$) is dependent upon the leaf area active in photosynthesis

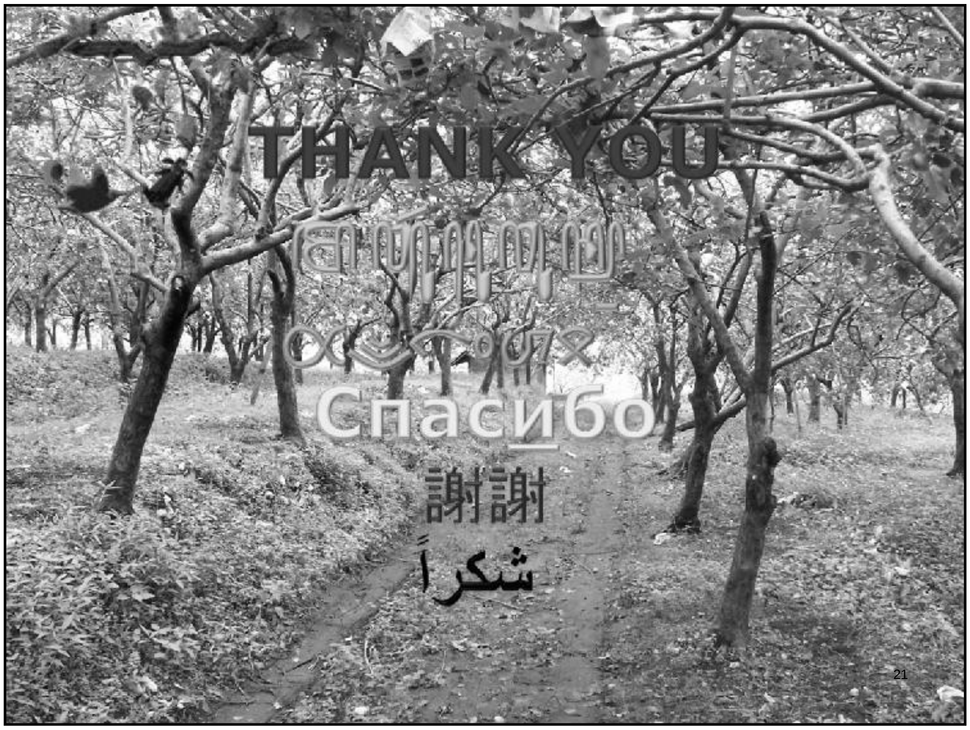
  $\partial W/\partial t) = \varepsilon LA$

  where ε is efficiency of leaf area in photosynthesis and widely known as NAR (net assimilation rate).

- Leaf area can be considered as the main part of plant machinery that determines the performance of plant system in the production of plant biomass.
- This assumption is different from the previous ones where plant machinery is represented by total dry weight (W) as an integration of leaf area, root density and metabolic processes.
- With the above assumption

$$\frac{\partial \mathbf{W}}{\partial \mathbf{t}} \frac{1}{\mathbf{L}} = \varepsilon = \mathbf{NAR}$$

$$\mathbf{NAR} = \frac{(\mathbf{W}_2 - \mathbf{W}_1)}{(\mathbf{T}_2 - \mathbf{T}_1)} \frac{(\ln \mathbf{LA}_2 - \ln \mathbf{LA}_1)}{(\mathbf{LA}_2 - \mathbf{LA}_1)}$$



dimana

ε = tingkat produksi biomassa per satuan luas daun yang juga dikenal dengan istilah harga satuan daun (HSD)

ε =E= ULR = NAR = HSD

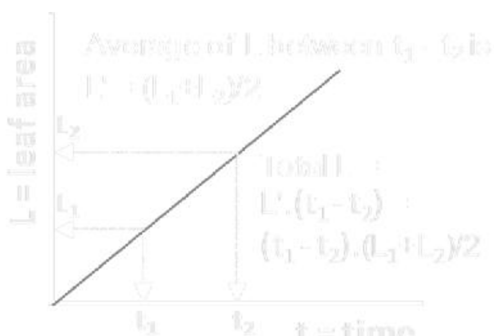
L = luas daun.

- Jika E (= ULR = NAR = HSD) dianggap konstan selama pertumbuhan tanaman atau selama masa tertentu yang dipertimbangkan, maka

$$\delta W = E.L\delta t$$

- Untuk bisa menggunakan persamaan ini,
  - total luas daun selama masa yang dipertimbangkan perlu diketahui dan
  - itu dapat diperoleh dari hubungan luas daun **L** dengan waktu **t** (umur tanaman)
- Luas daun total secara sederhana dapat diperoleh dari
  - harga rata-rata L (**L'**) dengan panjang masa yang dipertimbangkan dan
  - itu sama dengan integrasi dari luas daun selama masa tersebut seperti berikut.

$$L'(T_2 - T_1) = \int_{T_1}^{T_2} L\delta t$$



Sehingga

$$\delta W = E.L\delta t \Longrightarrow E = \frac{W_2 - W_1}{L'(T_2 - T_1)} \qquad \textbf{(12)}$$

dan untuk t = T_2-T_1

$$W = W_0 + \varepsilon L't \qquad \textbf{(13)}$$

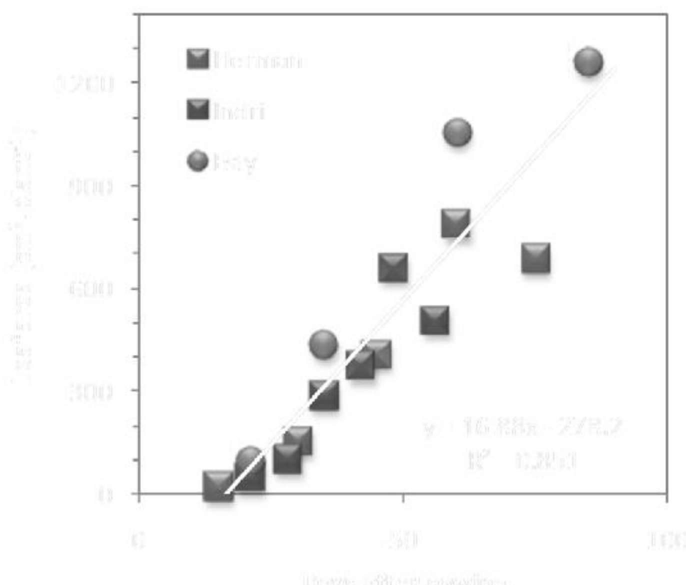
Asumsi 6.

- **Perkembangan luas daun dengan umur tanaman pada masa tertentu dapat konstan, sehingga hubungan L dengan waktu menjadi linier seperti berikut**

$$L = k + mt \qquad (14)$$

- **Dengan demikian, rata-rata luas daun selama masa yang dipertimbangkan adalah**

$$L' = \frac{1}{2}(L_1 + L_2) \qquad (15)$$



dan

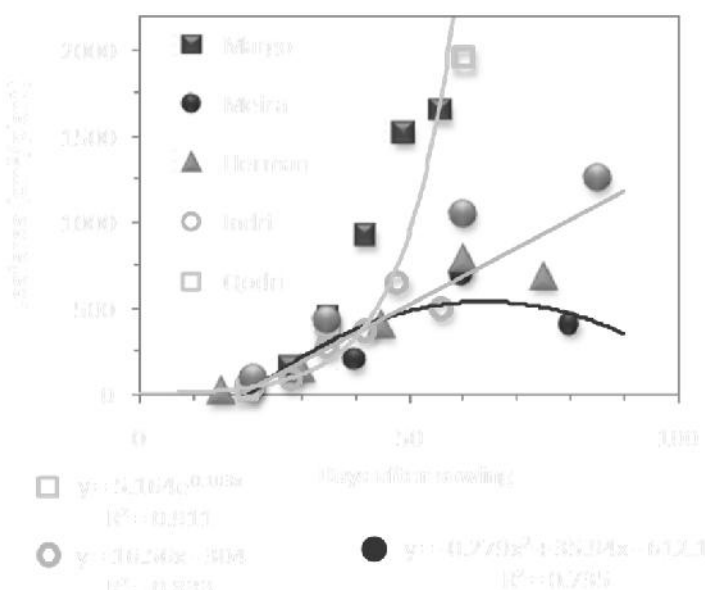
$$\mathbf{E} = \frac{2(\mathbf{W}_2 - \mathbf{W}_1)}{(\mathbf{L}_1 + \mathbf{L}_2)(\mathbf{T}_2 - \mathbf{T}_1)} \quad \textbf{(16)}$$

If t = T_2-T_1, n = 2 & 0 = 1, then

$$\mathbf{W} = \mathbf{W}_0 + 0.5(\mathbf{L}_0 + \mathbf{L}_\mathbf{n})\mathbf{E.t} \quad \textbf{(17)}$$

Compare with $\mathbf{W} = \frac{\mathbf{W_{max}}}{1 + \mathbf{a.e^{ut}}}$

This means that (L_0-L_1)E is not constant during the growth of plants



Asumsi 7.

- Perkembangan luas daun dapat terjadi secara eksponesil dengan umur tanaman pada masa tertentu yang dinyatakan dengan persamaan berikut.

$$L = pe^{qT} \quad (18)$$

- Dengan demikian, harga satuan daun adalah…….???→ *see Appendix*

$$E = \frac{W_2 - W_1}{T_2 - T_1} \cdot \frac{\ln L_2 - \ln L_1}{L_2 - L_1}$$

- This is the mostly widely used equation of NAR = ULR = E =HSD. This equation can be also derived from assumptions that of E is not constant and dry weight is linearly related to leaf area

If T2-T1 = t, then

$$(W_n - W_0)\ln(L_n/L_0) = Et(L_n - L_0)$$

$$(W_n - W_0) = \frac{Et(L_n - L_0)}{\ln(L_n/L_0)}$$

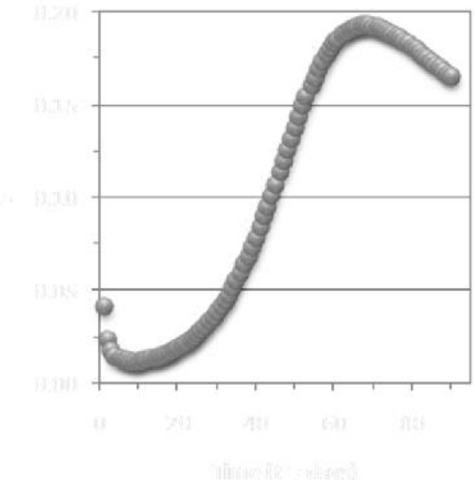
$$W_n = W_0 + \frac{Et(L_n - L_0)}{\ln(L_n/L_0)}$$

$$W_n = W_0 + \frac{(L_n - L_0)}{\ln(L_n/L_0)}Et$$

Compare with

$$W = \frac{W_{max}}{1 + a.e^{ut}}$$

This means that $(L_n\text{-}L_0)/\ln(L_n/L_0)E$ is not constant with time



$y = (L_n - L_0)/\ln(L_n/L_0)E$ or $y = 0.5(L_0 - L_1)E$

4. Bertalanffy curves

W=A*(1-b*exp(-ct))^3

$$W = A.\left(1 - b.e^{(-ct)}\right)^3$$

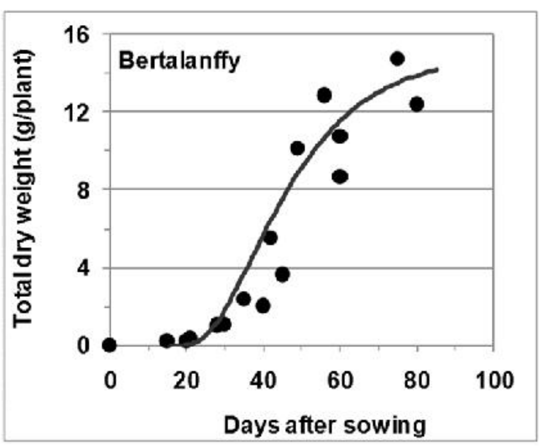
where A = asymptotic max, b & c = shape parameters

- This was developed by L.V.Bertalanffy in 1957 as a model for body weight growth
- The point of inflection is fixed at 8/27 or 29,63% of the max value
- Suitable for sigmoid growth with inflection points around 30% of the ultimate

Ref:Bertalanffy,L.V.(1957):*Wachstum.In:Helmcke,J.G.,H.v.Len-GerkenundG.Starck(Ed.):HandbuchderZoologie.Berlin:W.deGruyter,Bd.8,10.Lieferung,1-68*

$$W = A.\left(1 - b.e^{(-ct)}\right)^3$$

Wmax	15
b	3.0269
c	-0.0599



5. Brody curves

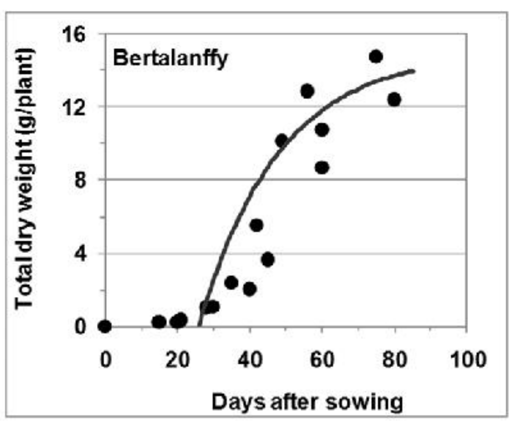
W=A*(1-b*exp(-ct)) $\mathbf{W} = \mathbf{A}.\left(1 - \mathbf{b}.\mathbf{e}^{(-\mathbf{ct})}\right)$

Where the inflection points occurs between the two curves

- Derived by S.Brody as models for piecewise growth processes of exponential type
- The increasing unction is only valid in temporarily limited intervals and cant be extrapolated
- The decreasing function can be extrapolated and is suitable for monotonous decrease

Ref: Brody, S. (1945): *Bioenergetics and growth. New York: Reinhold*

$$\mathbf{W} = \mathbf{A}.\left(1 - \mathbf{b}.\mathbf{e}^{(-\mathbf{ct})}\right)$$



Wmax	15
b	3.228
c	-0.0454

6. Janoschek models

W=A*(1-exp(-b*t^c)

- This was developed by A. Janoschek in 1957 as a model for reaction kinetic parameters
- Similar to Richard curves and a generalization of more specialized curves
- Rarely convergence problems and easier to work with than Richard models

Ref: Janoschek, *A. (1957): Das reaktionskinetische Grundgesetz und seine Beziehungen zum Wachstums-und Ertragsgesetz. Stat. Vjschr. 10, 25-37*

Appendix

- $LD_1 = pe^{qT_1} = p.\exp(qT_1)$
- $LD_2 = pe^{qT_2} = p.\exp(qT_2)$

$$\frac{\mathbf{LD}_2}{\mathbf{LD}_1} = \frac{\mathbf{p}.\exp(\mathbf{qT}_2)}{\mathbf{p}.\exp(\mathbf{qT}_1)}$$

- $LD_2/LD_1 = \exp[q(T_2-T_1)]$
- $\ln(LD_2/LD_1)= [q(T_2-T_1)]$
- $\ln(LD_2) - \ln(LD_1) = q(T_2-T_1)$
- $q = [\ln(LD_2) - \ln(LD_1)]/(T_2-T_1)$

- LD = p.e^{qT}

$$\int_{T_1}^{T_2} LD.\delta T = p\int_{T_1}^{T_2} e^{qT}.\delta T \qquad \int_{T_1}^{T_2} LD.\delta T = \frac{p}{q}\left(e^{qT_2} - e^{qT_1}\right)$$

- $p.e^{qT_2} = LD_2$ and $p.e^{qT_1} = LD_1$
- $LD = (LD_2 - LD_1)*(T_2-T_1)/[\ln(LD_2) - \ln(LD_1)]$
- Hence

$$E = \frac{W_2 - W_1}{T_2 - T}.\frac{\ln LD_2 - \ln LD_1}{LD_2 - LD_1}$$

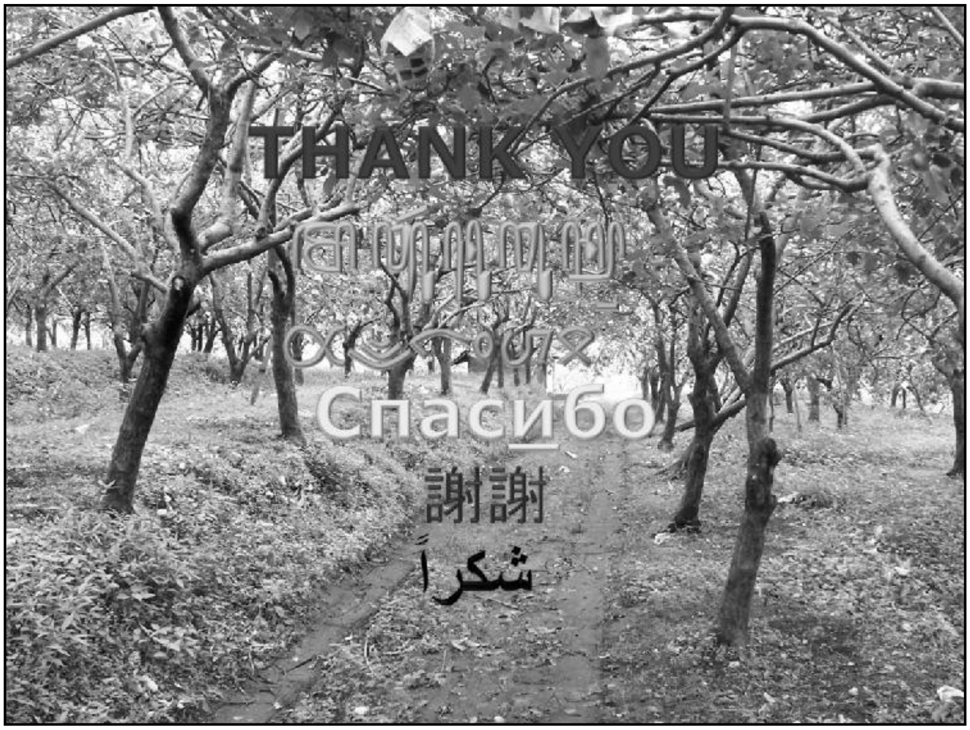
- This is the mostly widely used equation of NAR = ULR = E =HSD

$$E = \frac{W_2 - W_1}{T_2 - T}.\frac{\ln LD_2 - \ln LD_1}{LD_2 - LD_1}$$

$$W_2 - W_1 = E\frac{LD_2 - LD_1}{\ln LD_2 - \ln LD_1}.(T_2 - T_1)$$

$$W_t - W_0 = E\frac{LD_t - LD_0}{\ln LD_t - \ln LD_0}.(T_t - T_0)$$

$$W_t = W_0 + \frac{L_t - L_0}{\ln(L_t / L_0)}Et$$


THANK YOU
Спасибо
謝謝
شكراً